Final Exam-MATH 413-(Complex Analysis)

Saturday 25\2\1435H 1st Semester – 2 Hours

Dr. Abeer Badghaish

Name:____Co:_____________________

Q1# Answer True or False:

1- $\mathrm{Log}(z) = \ln|z| + i\theta$, $(-\pi < \theta < \pi)$ is the Principle branch of log(z)..........( )

2- A function that is analytic for all $z \in C$ is called singular function.....................( )

3- If $z \in C - \{0\}$, then $e^{\log z} = z$ for any value of the function log z...............( )

4- $|i^3| = i$ ....( )

5- $f(z) = \frac{1}{1-z^2}$ has Maclaurin series given by $f(z) = 1 + z^2 + z^4 + \cdots$,...........( )

6- z_0 is an isolated singular point if f is analytic in the deleted neighborhood of the point z_0...( )

7- $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$...( )

8- If a function is analytic through a simply connected domain D, then for every closed contour C lying in D, $\int_C f(z)dz = 2\pi i$...( )

Q2# Fill the blanks

1- The n^{th}root of $z_0 = r_0 e^{i\theta_0}$ is $z =$ ________________________________.

2- If $z = 1 + i$ then $z^{-1} =$ ______________________________

3- The function $f(z) = \frac{1}{z^2+3}$ has two singular points at $z =$ ______ and $z =$ _______

4- If $\lim_{z \to z_0} f(z) = L$. Then $\lim\limits_{z \to z_0} |f(z)| =$ ____________.

5- An isolated singular point z_0 of a function f is a pole of order m iff $f(z)$ can be written as $f(z) =$ ____________________, where $\emptyset(z)$ is analytic nonzero at z_0. Moreover, for $m \geq 2$, $\mathrm{Res}_{z=z_0} f(z) =$ ______________________.

6- Let C be a simple closed contour in positive sense. If $f(z)$ is analytic inside and on C except for finite number of singular points $z_k(k = 1,2, \ldots n)$ inside C, then

$$\int_C f(z)dz = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

7- Let two functions p and q be analytic at a point z_0. If $p(z_0) \neq 0,\ q(z_0) = 0,$ $q'(z_0) \neq 0$, then z_0 is a simple pole of $^p/_q$ and

$$\mathrm{Res}_{z=z_0} \frac{p(z)}{q(z)} = \_\_\_\_\_\_\_\_\_\_\_\_$$

<u>Q3#</u> For $\mathrm{z = x + iy}$ show that if the function $f(z)$ satisfy Cauchy-Riemann equations or does not satisfy it:

a- $f(z) = e^{-x}(x \sin y - y \cos y\,)$, **b-** $f(z) = Im(z)$

<u>Q4#</u> The following functions has singular point at $z = 0$

$$\boldsymbol{f(z) = \frac{1 - \cos z}{z^2}}, \qquad \boldsymbol{g(z) = \frac{\sin z}{z}}$$

a- Expand the functions about $z = 0$

b- What is the type of the singular point $z = 0$

Q5# Find the series expansion of the function

$$f(z) = \frac{1}{z-3}$$

a- If $|z| < 3$,

b- If $|z| > 3$.

Q6# Show that the function $f(z) = \frac{1}{z}$ has the series expansion

$$\frac{1}{z} = -\sum_{k=1}^{\infty} \frac{(z+2)^k}{2^{k+1}} \quad , \quad if \quad |z+2| < 2$$

<u>Q7#</u> Find the **Residue** of the following functions at $\boldsymbol{z=0}$:

a- $\boldsymbol{f(z)=\frac{z^2+1}{z}}$ [**Hint**: you can use the theorem in **Q2**(7)]

b- $\boldsymbol{f(z)=\frac{z^3+i}{z^4}}$ [**Hint**: you can use the theorem in **Q2**(5)]

Q8# Evaluate

$$\int_C \frac{z+1}{z^2-2z}\,dz$$

where C is the circle $|z|=3$ [**Hint**: you can expand the function about its singular points, then use the theorem in **Q2**(6)]